THE DEMONSTRATION CORNER

NOT-SO-SIMPLE
HARMONIC OSCILLATOR

by

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Two years ago when I was in The Netherlands for the International Physics Olympiad, the Soviet team-leader, Sergey Krotov[1], demonstrated a remarkable toy, crafted to the highest standards by the technical staff at Moscow State University. It consisted of a series of simple pendulums of varying lengths which, when swung together, formed very beautiful patterns. I built one of my own which works just as well, using only bits and pieces that I found in my high school physics lab.

I used a half-metre ruler, 12 bolts, washers and wing-nuts, 11 steel balls (about 2 cm diameter), string, duct tape, and a C-clamp, to build a contraption that looks like this:
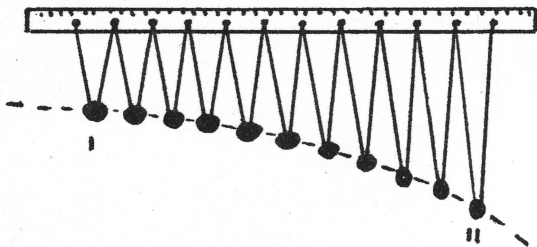
A piece of duct tape encircles each ball and a length of string passes under the duct tape and is attached to the half-metre rule with the bolts, washers and wing-nuts through holes drilled for the purpose. Hence, each ball is supported in such a way that it will swing with no elliptical motion; the length of its supporting string may be easily changed by loosing the appropriate wing-nut. I support the whole thing on any handy shelf or table leg with the C-clamp.

The holes were drilled 4 cm apart and the first bob was hung 12.5 cm ($=\ell_1$) below the rule. The bob on the longest string, label it the 11th bob, was hung 25 cm ($=\ell_{11}$) below the ruler. From the formula for the period of a simple pendulum,

$$T = 2\pi\sqrt{\frac{\ell}{g}} \quad ,$$

the period of the short bob is $T_1 = 0.7$ s whereas the long bob's period is about $T_{11} = 1.0$ s. The remaining bobs are equally spaced and have lengths that satisfy the parabolic equation $\ell = kx^2$, where I used $k \approx 0.001$ cm^{-1}. Simple substitution gives for the first bob, $x_1 = 112$ cm. The x-value for the second bob is larger by 4 cm and so on.

What is so special about this toy? Clearly the length of any bob obeys $\ell \propto x^2$ and its period follows $T^2 \propto \ell$ so that we have $T \propto x$. Then ΔT ($\propto \Delta x$) is the difference in period between any two adjacent bobs. Since the bobs are equally spaced, each

bob has a period which is a constant ΔT longer than its adjacent shorter pendulum. Differentiating $T = 2\pi\sqrt{kx^2/g}$ gives:

$$\Delta T = 2\pi\sqrt{\frac{k}{g}}\Delta x \quad .$$

Substitution gives $\Delta T = 27$ ms.

When the bobs are all displaced a certain amount together and released, they will start swinging in phase with one another but will very quickly deviate from this symmetry. At a least common multiple of the periods of all the bobs, they will be back in phase again. For my device, this occurs after the long bob has executed 26 periods. The short bob will have done 36 periods. We can write that $26T_{11} = 36T_1 = 35T_2 = 34T_3$, and so on.

From this we see that after 13 swings of the long bob, every second bob is in phase. Generally, at any time that any two bobs are in phase with each other, a sine-wave pattern is formed by the toy. (If we had a very great number of very small bobs spaced very closely, there would always be at least two bobs in phase...but we are limited by our time and materials.) Successive snapshots of the toy in action, as viewed from below, might look as follows:
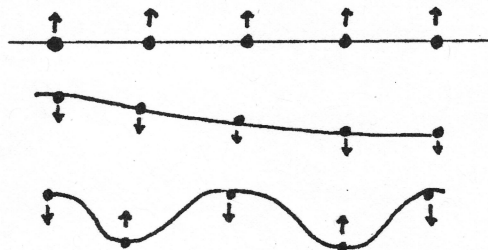
The wavelength of the sine wave shrinks as the oscillations develop and then expands again as the first and last bobs come back into phase. The tricky part of the construction is the fine tuning of the lengths of the strings. After calculating approximate lengths, I spent almost an hour adjusting them bit by bit, bringing each bob into character. As you might expect, the shorter ones are the trickiest since they are the least "simple" and their true period deviates slightly from that given in our formula.

The toy, apart from being useful in demonstrating the physics of a simple pendulum and beating is almost a work of art. Once you have one, you will be looking for ways to show it off to your students.

[1] Sergey Krotov is the editor of Kvant Magazine published by the Soviet Academy of Sciences. An English-language version, called Quantum, is now published by the NSTA in the U.S.A., and should be on every physics and math teacher's reading list.